Multiresolution image VQ compression by color codebook reordering

Christophe Charrier   Olivier Lezoray

Université de Caen-Basse Normandie, GREYC UMR CNRS 6072, Equipe Image
120, rue de l'exode, 50000 Saint-Lô, France – Phone : +33 (0)2 33 77 55 11 - Fax : +33 (0)2 33 77 11 67
E-mail: {`christophe.charrier,olivier.lezoray`}`@unicaen.fr`

Abstract

In this paper, a method to reach multiresolution VQ is proposed. A reordering process of color vectors from the codebook is performed. This yields to obtain an index image very close to the initial luminance image that can be obtained from any color image, but at a lower resolution. That way, the VQ technique can be applied once again on this new index image. In other words, a hierarchical and multiresolution VQ is defined. At each stage of the VQ compressed image, one is able to reconstruct it using the initial color codebook and the index one.

A comparison of the proposed method and classical Hierarchical VQ (HVQ) scheme at different compression levels is performed to judge the performance and the robustness of the hybrid method. B oth objective and subjective measures are used.

This work is a preliminary work to compress high dimension images at different levels.

Introduction

From all existing lossy compression techniques, the Vector Quantization (VQ) technique has been widely used in the last two decades. This technique is a generalization of the scalar quantization process to the quantization of a vector belonging to a set in which one partial order is provided at least. This compression technique has been selected among others since theoretically no other coding technique can do better than VQ. Actually, we enumerate the set of binary words produced by the coding system as indexes $1, 2, \ldots, N$. For the ith binary word, let the decoded output of the given coding system be vector y_i. Then, a VQ decoder achieves equivalent performance to the decoder of any given coding system and a VQ encoder can be defined to be identical to the encoder of any given coding system [1]. The higher the dimension of vectors and the fewer the number of indexes, the higher the compression rate. Nevertheless, for high dimension vectors, the reconstructed images usually suffer from visible block structure.

Various improvements of the VQ techniques have been developed in order to achieve the best trade-off between compression rate and reconstructed image quality. Two main approaches have been investigated : 1) the reduction of the address bits and 2) the reduction of the bit rate of smooth regions [2]. In the first approach, one tries to design an optimal subset of the codebook by evaluating the correlation of the adjacent pixels across block boundaries. A typical technique is the finite state vector quantization. The second approach is based on the emphazis of the features of the different regions in an image, such as the variable block vector quantization, and the hierarchical multirate vector quantization where the blocks with different grayscale transition features are assigned into different layers.

One way to reach the optimal quality-compression rate tradeoff is the development of hierarchical techniques for the design of vector quantizer (VQ) encoders implemented by table lookups rather than by a minimum distortion search [2]. In a table lookup encoder, input vectors to the encoder are used directly as addresses in code tables to choose the channel symbol codewords. In order to preserve manageable table sizes for large dimension VQs, hierarchical structures are used to quantize the signal successively in stages. The encoder of a hierarchical VQ (HVQ) consists of several stages, each stage being a VQ implemented by a lookup table. Since both the encoder and the decoder are implemented by table lookups, there are no arithmetic computations required in the final VQ implementation [3].

In such a way, hierarchical does not mean multiresolution. In this paper, a new method to reach multiresolution VQ is presented. The main idea is to reorder color vectors from the codebook to obtain an index image very close to the initial luminance image that can be obtained from any color image, but at a lower resolution. In that case, the VQ technique can be applied once again on this new index image. That way, a hierarchical and multiresolution VQ is defined. At each stage of the VQ compressed image, one is able to reconstruct it using the initial color codebook and the index one. This global scheme is shown in Fig. 1.

Specification of the VQ

VQ maps a vector $\mathbf{x}$ of dimension $\mathbf{k}$ to another vector $\mathbf{y}$ of dimension $\mathbf{k}$ that belongs to a finite set $\mathbf{C}$ (*codebook*) of output vectors (*codewords*). Thus, the vector quantizer $\mathbf{Q}$ is defined as follows

$$\mathbf{Q} : \mathbb{R}^k \to \mathbf{C}, \tag{1}$$

where $\mathbf{C} = (y_1, y_2, \ldots, y_N)$ and $y_i \in \mathbb{R}^k, \forall i \in \mathbf{I} \equiv \{1, 2, \ldots, N\}$.

The vector quantizer $\mathbf{Q}$ can be decomposed into a vector encoder operation ε and a vector decoder operation $\mathbf{D}$. The encoder $\varepsilon : \mathbb{R}^k \to \mathbf{I}$ is a mapping from $\mathbf{R}^k$ into an index set $\mathbf{I}$. The decoder $\mathbf{D} : \mathbf{C} \to \mathbb{R}^k$ maps the index set $\mathbf{I}$ into the codebook $\mathbf{C}$.

Thus the overall operation of the VQ can be viewed as the composition of these two operations:

$$Q(\mathbf{x}) = \mathscr{D}.\varepsilon(\mathbf{x}) = \mathscr{D}(\varepsilon(\mathbf{x})). \tag{2}$$

Associated with every $\mathbf{n}$ point vector quantizer is a *partition* of $\mathbb{R}^k$ into $\mathbf{n}$ regions or *cells*, $\mathbf{R}_i$ for $i \in \mathbf{I}$. The ith cell is defined
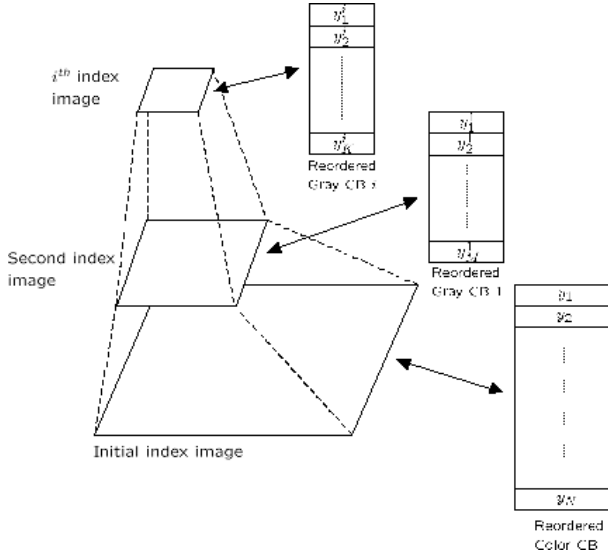
Figure 1. *The proposed multiresolution and hierarchical VQ-based compression scheme.*

by

$$\mathbf{R}_i = \{x \in \mathbb{R}^k : Q(x) = y_i\}. \tag{3}$$

To find the color space which best minimizes distortion, the Minkowski formula can be used to determine the distance between two codewords:

$$d(x,y) = \left(\sum_{i=1}^{k} (x_i - y_i)^r\right)^{1/r}. \tag{4}$$

where $r \in [0, \infty)$. Although this distance is not based directly on perceptual principles, the most commonly used to generate codebooks is the L_2 norm obtained for $r = 2$.

Reordering process

There are many techniques to reorder color palettes [4, 5]. As palette reordering is a class of preprocessing methods aiming at finding a permutation of the color palette such as the resulting image of indexes is more amenable for compression. This is always done for scalar quantization. We propose to apply such a technique to vector quantization. Instead of working with single pixels, the reordering process will be done with a set of pixels, labelled as vectors.

To reorder the images of VQ indexes two main approaches can be considered: 1) vector based technique and 2) index based techniques [2]. For the latter, used methods analyze the image statistics, and construct a mapping of reduced variance based on these statistics. Nevertheless, these methods are deeply linked to the content of the image and a mapping table needs to be transmitted for each image with the information to reverse the reordering. Vector-based methods try to approximate the distribution to that of the original image which is known to be smooth (for natural images) and with the intendend laplacian distributed differences. It only depends on the used codebook.

To reorder the elements of a vector color palette, a spectral decomposition of the random walk transition matrix is computed [6]. Let P denote the transition matrix, D the degree matrix and S a similarity matrix, one has $P = D^{-1}.S$. This is obtained by

considering a complete graph built over the color palette. The spectral decomposition of P is $P = \sum_{i=1}^{N} \lambda_i v_i v_i^T$ where λ_i are the eigenvalues and v_i the eigenvectors. Once this decomposition is obtained, the color palette is reordered according to the value of the second highest eigenvector. This corresponds to the Laplacian Eigenmaps principle [6] used here to map an input space $\mathbb{R}^k$ to $\mathbf{I}$.

Let $\{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_n\} \in \mathbb{R}^k$ be n sample vectors. Given a neighborhood graph G associated to these vectors, one considers its adjacency matrix W where weights W_{ij} are given by a Gaussian kernel $W_{ij} = k(\mathbf{x}_i, \mathbf{x}_j) = e\left(-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{\sigma^2}\right)$. Let D denote the diagonal matrix with elements $D_{ii} = \sum_j W_{ij}$ and Δ denote the unnormalized Laplacian defined by $\Delta = D - W$. Laplacian Eigenmaps dimensionality reduction consists in searching for a new representation $\{\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_n\}$ with $\mathbf{y}_i \in \mathbb{R}^n$, obtained by minimizing:

$$\frac{1}{2}\sum_{ij} \left\|\mathbf{y}_i - \mathbf{y}_j\right\|_2 W_{ij} = Tr(\mathbf{Y}^T \Delta \mathbf{Y})$$
$$\text{with } \mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_n].$$

This cost function encourages nearby sample vectors to be mapped to nearby outputs. This is achieved by finding the eigenvectors $\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_n$ of matrix Δ. Dimensionality reduction is obtained by considering the q lowest eigenvectors (the first eigenvector being discarded) with $q \ll p$. Therefore, we can define a dimensionality reduction operator $h : \mathbf{x}_i \rightarrow (y_2(i), \cdots, y_q(i))$ where $y_k(i)$ is the ith coordinate of eigenvector $\mathbf{y}_k$.

The hybrid method

In this paper, a hybrid method is used to construct a hierarchical and multiresolution VQ scheme. At the first stage of the process, an initial color image is decomposed in K_1 blocks. Then, the classical VQ scheme is performed. At this level, an index image I_1 is obtained as well as its associated Color Code-Book (CCB) of size N. Then the reorder process is applied to color codebook to generate a VQ index image that is very close to the inverted luminance image as shown in Figure 2).

To obtain the second level of the pyramid as depicted in Fig. 1, a new codebook is constructed from this "luminance" image. At this second level, a new VQ index image I_2 of size $K_2 < K_1$ is created as well as a new Gray CodeBook GCB_2 of size M. This new codebook contains indexes used within the first VQ index image. If necessary, a new color codebook can be obtained from the initial color codebook and the obtained gray one. Actually, each index included in the code vector of the gray codebook yields us to reach the associated color code vector of the initial color codebook. In that case, one can build a new color codebook CCB_1 that can be interpreted as a the initial color codebook computed for a VQ compression based on a decomposition of the initial image in K_1 blocks.

At this stage, to construct the third level of the pyramid, one has to reorder the gray level codebook GCB_2 to obtain a new "luminance image". Then, one can design a third VQ index image I_3 of size $K_3 < K_2$ and a new gray codebook GCB_3. And so on.

Thus, at each step, one is able to design a VQ index image as well as its associated color codebook.
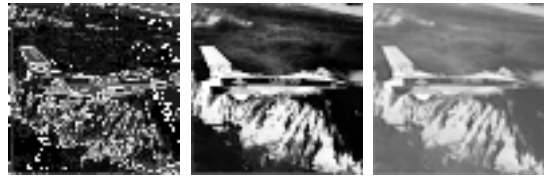
(a) Initial VQ index image (b) Reordered VQ index image (c) Original inverted luminance image
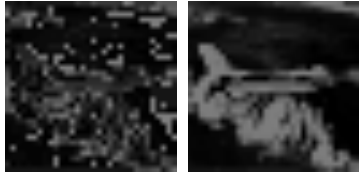
(d) VQ index image at the 2nd level (scale=2) (e) Reordered VQ index image at the 2nd level (scale=2)

Figure 2. *Rought initial VQ index image (a) and reordered VQ index image (b) both obtained at the first level of the pyramid. Image (b) is very close from the inverted luminance image (c). Images (d) and (e) respectively represent the obtained rought VQ index image and reordered VQ image at the second level of the pyramid.*
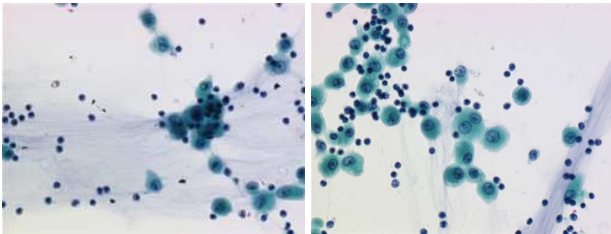
Figure 3. *Samples of the used images database.*

Measure of performance

In order to judge the performance and the robustness of the proposed hybrid method, a comparison with classical VQ schemes (Hierarchical, VQ) at different compression levels is performed.

Hierarchical Vector Quantization is a particular VQ for which the quantization of an image vector is broken up into a series of quantizations on subcomponents of the vector, and in turn the indices resulting from each pair of those quantizations can be fed into another quantization, cascaded until the overall image vector quantizes to a single index. This series of quantizations are managed such that each of the individual quantization steps can be precomputed, and performed using a single LUT. The HVQ encoding process then becomes a series of LUTs cascaded. This results in an extremely fast encoding algorithm, since no computation is required.

The proposed multiresolution color reordered technique has been applied to an image database containing 50 microscopic image. Figure 3 shows an example of that database.

Objective measures

To assess the performance of the proposed technique, we have to judge the quality of the reconstructed image at each generated level. We propose to use three different segmentation error metrics: the MAE (Mean Average Error), the PSNR (Peak Signal to Noise Ratio) and the MSE (Mean Square Error) [7]. They are

defined by the following expressions, where I and J denote two color images of size $h.w$:

$$MAE(I,J) = \frac{1}{3.h.w} \sum_{k=1}^{3} |I_{C_k} - J_{C_k}| \qquad (5)$$

$$MSE(I,J) = \frac{1}{3.h.w} \sum_{k=1}^{3} \|I_{C_k} - J_{C_k}\| \qquad (6)$$

$$PSNR(I,J) = 10log_2 \left(\frac{255^2}{MSE(I,J)} \right) \qquad (7)$$

where I_{C_k} and J_{C_k} respectively represent the kth color channel of the image I and J, and $|.|$ and $\|.\|$ respectively denote the L_1 and L_2 norms.

Subjective measures

Psychophysical measures of quality let us define the sensitivity threshold of two image differences. This threshold characterizes the standard subject sensitivity. This sensitivity is performed using a psychophysical approach of performance measures based on detection theory.

To determine the sensitivity of the subject, we used the a forced-choice experiment [8]. This sensitivity measure is defined by the two probabilities $p(H)$ (Hit) and $p(CR)$ (Correct Rejection). This measure d' simply represents the distance between the mean of the distribution of the <first stimulus> and the <second stimulus> under the Gaussian distribution assumption of each stimulus.

Nevertheless, d' does not characterize the method but only the stimuli pair. Under the assumption of unbias responding, the sensitivity measure d' can be expressed as follows [8]:

$$d' = 2z[\frac{1}{2}(1 + \sqrt{2p(c) - 1})], \qquad (8)$$

where $p(c) = (p(H) + p(CR))/2$ and $z[.]$ is the inverse of the normal distribution function.

The differences between the two stimuli are imperceptible if d' is less than 0.5. Between 0.5 and 1.0, differences between the two stimuli are just noticeable. If d' is greater than 1.0, the two stimuli are more and more different.

Psychophysical Experiments

Observers included several subjects who were naive about the aims of the experiment and who have normal color vision. This has been evaluated thanks the well known Ishihara test.

Observers evaluated pairs of reconstructed VQ images <VQ1,VQ2>. VQ1 represents the reconstructed image encoded using the proposed method, whereas VQ2 is the image encoded using the HVQ principle.

The question asked to the observers was: "Where is the best image in terms of quality ?". The two images were presented to each observer, with the first one placed randomly on the right in half of the presentations. 160 presentations of each pair were used to obtain a large number of responses and moreover to avoid bias responses. Using such a number, we obtain a more robust value of the observer's sensitivity.

Results

Figure 4 shows the obtained results for the three used metrics. For each one of those three metrics, one can observe that the

obtained quality for the reconstructed hybrid VQ-based image is higher than the one obtained using the HVQ algorithm. This is mainly due to the fact that using the proposed method one can rebuild the color codebook at a given decomposition level from the initial color codebook. In that case, color that are less representative can be conserved in the final codebook. This can not be the case when color equivalent codebooks are directly generated at the given level.

Nevertheless, it is well known that those kinds of measure do not exactly reflect the human judgment, in that sense, that many internal scales are requested to get to the final judgment. A simple average of the whole implied scales is not performed, that is not the case when using objective metrics.

To assess the previous obtained results, one use psychovisual tests as described in section . Table 1 shows the obtained results of a such test from a typical observer.

Stimuli	VQ2 on left	VQ2 on right
<VQ2,VQ1>	65	15
<VQ1,VQ2>	18	62

Example of the response from a typical observer at the third level of the pyramid decomposition.

The sensitivity value obtained is about 1.1. This result can be interpreted as follows: observers could frequently distinguish the reconstructed image using the proposed approach from the reconstructued one usign the HVQ technique; the first one is so preferred to the second one even if differences are just noticeable.

To estimate the evolution of that sensitivity value with respect to the decompoistion level, we calculate the observer's sensitivity at each decomposition level. Results of these calculations are shown in figure 5.

One can see that the higher the decomposition, the greater observers' sensitivity. Nevertheless, even if that measure is higher than 1, one can not conclude that the reconstructed hybrid VQ-based image quality is indeniably better than the other one. One can just argue that the differences between the two images are more and more noticeable as the decomposition level increases. This is really interesting for high dimension images, such as microscopic images, since the number of decomposition level can easily reaches 6 or 7. In that case, we can predict that the quality of reconstructed hybrid VQ-based image will be higher that the other one. An example of the reconstructed image using the hybrid VQ encoding process through 3 different levels of the pyramid is shown in Figure 6. One can observed that at the last level, the global structure of the original image is preserved. The size of the used original image is 756×564.

Conclusion

In this paper, a multiresolution approach to design the codebook at each level of the hierarchical structure is presented. The proposed scheme is based on a color reordering process applied to the initial color codebook. This is performed using the Laplacian Eigenmaps principle to map the initial color space to the index set. That reordering process applied to color codebook yields to generate a VQ index image that is very close to the inverted luminance image.
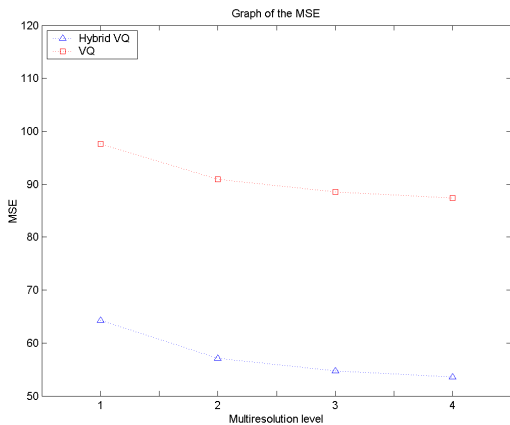
This is a preliminary work. Future works are concerned
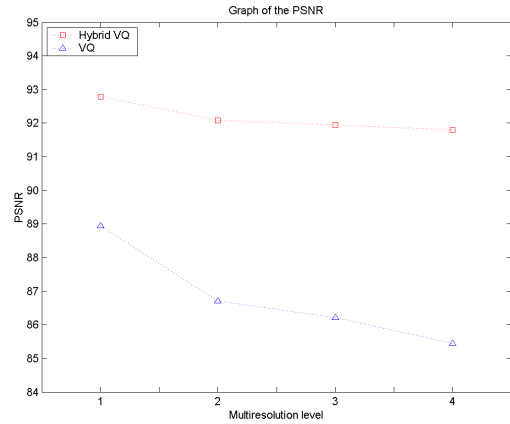
(a) PSNR measure
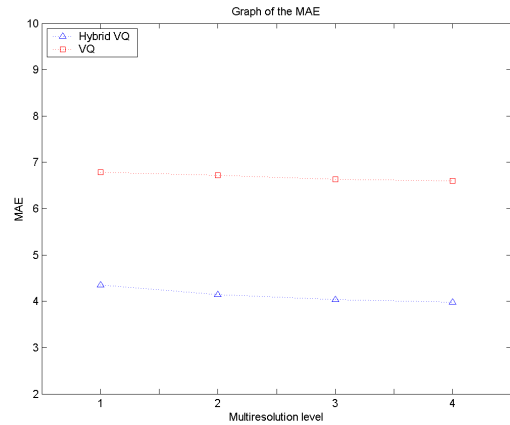
(b) MAE measure

(c) MSE measure

Figure 4. *Objective measures of the quality of the reconstructed images for both classical HVQ and the proposed method.*
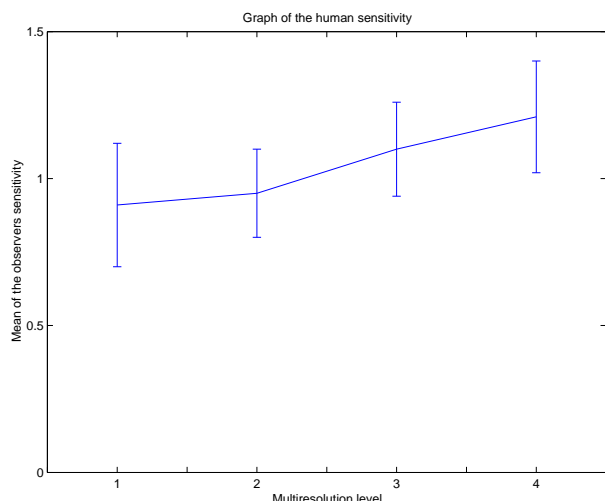
**Figure 5.** *Evolution of the human sensitivity measure with respect to the decomposition level.*

by applying the developed technique on high dimension images, such as biomedical images to help the segmentation process for such kind of images.

Acknowledgments

References

[1] A. Gersho and R. M. Gray, *Vector Quantization and Signal Compression*. Kluwer Academic Publishers, 1991.

[2] P. Yu and A. N. Venetsanopoulos, "Hierarchical finite-state veector quantization for image coding," *IEEE Transactions on Communications*, vol. 42, pp. 3020–3026, Nov. 1994.

[3] A. R. C. Paiva and A. J. Pinho, "Evaluation of some reordering techniques for image vq index compression," in *ICIAR 2004*, vol. LCNS 3211, (Porto, Portugal), pp. 302–309, Sept. 2004.

[4] A. J. Pinho and A. J. R. Neves, "A survey of palette reordering methods for improving the compression of color-indexed images," *IEEE Transactions on Image Processing*, vol. 13, pp. 1411–1418, Nov. 2004.

[5] S. Battiato, F. Rundo, and F. Stanco, "Color palette images re-indexing by self organizing motor maps," in *Eurographics Italian Chapter*, pp. 22–28, 2006.

[6] M. Belkin and P. Niyogi, "Laplacian eigenmaps and spectral techniques for embedding and clustering," in *Neural Information Processing Systems*, (Vancouver, BC), pp. 585–591, Dec. 2001.

[7] K. N. Platoniotis and A. N. Venetsanopoulos, *Color Image Processing and Applications*. Springer, 2000.

[8] N. A. Macmillan and C. D. Creelman, *Detection theory: a user's guide*. Cambridge University Press, 1990.
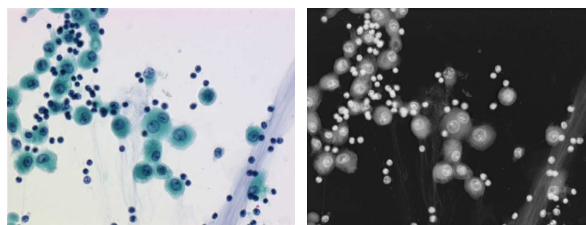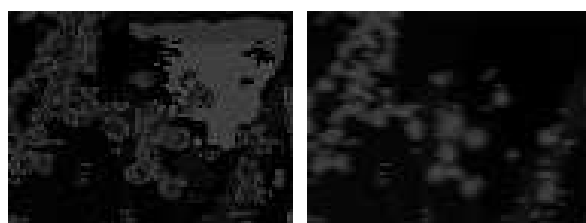
(a) Original microscopic image

(b) Original inverted luminance image
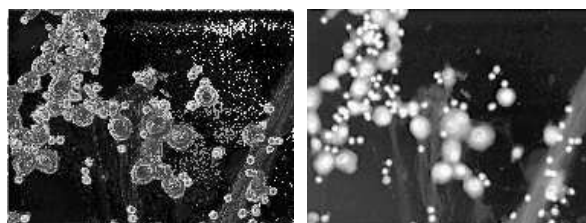
(c) VQ index image at level 3

(d) Reordered VQ index image at level 3
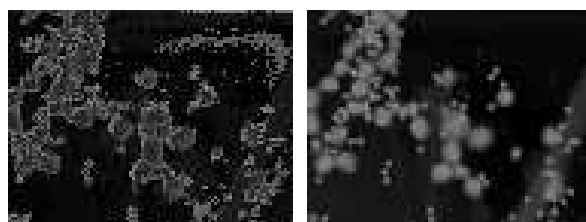
(e) VQ index image at level 5

(f) Reordered VQ index image at level 5

(g) VQ index image at level 7

(h) Reordered VQ index image at level 7

Figure 6. *Obtained rought VQ index images (c,e,g) and reordered VQ images (d,f,h) at the three different levels of the pyramid. Image (b) represents the initial inverted luminance image of the original image (a)*